



## The Math Forum: Problems of the Week

# *Problem Solving and Communication*

## *Activity Series*

## Understanding the Problem

What does it mean to fully understand a problem, and how does it help students find solution paths and build confidence? Understanding the problem is the first principle of problem solving described in mathematician George Polya's (1945) *How to Solve It*. It is also related to math educators Stephen Krulik and Jesse Rudnick's (1987) first problem-solving stages, read and explore, described in their book, *Problem Solving: A Handbook for Teachers*. Included in this document are several activities that support students to develop strategies for understanding challenging math problems, along with facilitation suggestions for teachers.

### Goals for the Understanding the Problem Strategy

Different techniques for understanding the problem can lead us to ideas for solving a problem we have never used before. Good problem-solvers use this problem-solving strategy and may come back to it often as they're working on the problem, to refine their strategy, see if they can find better solutions, or find other, even more interesting questions. Specifically, good problem-solvers:

- Use various methods to make sense of a problem from different perspectives.
- Pull out the relevant information.
- Figure out what counts as an answer.
- Connect to prior knowledge and experience.
- Identify questions and relationships that make the problem interesting.

### Writing Goals

Writing is an integral part of understanding the problem and builds momentum in thought. It helps the problem solver organize their information and articulate the questions they will address on their journey towards the solution. Specifically, when trying to understand the problem, good problem-solvers might write:

- Create an organized list of what they noticed and what they wondered about the problem.
- Paraphrase the problem in their own words.
- Question or hypothesize, focusing their thinking on specific parts of the problem (e.g., I'll know I'm right when... ; I could solve the problem if ...; the part that makes it hard is ...; I'll need to use the fact that ...)

## Activities

### *I. Noticings/Wonderings*

**Format:** pairs/small groups, big-group brainstorming or go-round activity, or think-pair-share.

Make a list of all of the mathematical information and relationships you notice about the problem, and everything you wonder about the problem. Your noticings may include:

- The quantities that can be counted or measured.
- Relationships between quantities.
- Information that is not given in the problem.
- Key words from the problem.

Your wonderings may include:

- I wonder what will happen if ...
- I wonder what this word means ...
- I wonder if this pattern will continue ...

### Sample Activity: Forget the Question

Present the problem to students without the question. As a whole class, or in small groups, have students go around quickly and offer one mathematical relationship or bit of mathematical information that they notice, or a mathematical question they are wondering about. Record each noticing and wondering for later use. Continue around the group until no one has anything more to offer.

**Note:** It is useful to refer back to and update your class list of noticings/wonderings throughout the process of understanding the problem and solving the problem. Students should be encouraged to keep their list, and/or the class list, available and update it as they work.

#### Key Outcomes

- Student ownership and understanding of the question to be solved.
- Momentum toward a solution path stimulated by all of the mathematical quantities and relationships noticed.
- Slowing down the thinking process and surfacing all of the information and questions that are too easily passed over or dismissed.
- Articulation of specific sub-problems or questions students need to answer or learn more about in order to solve the problem.
- Identifying other questions and features of the problem that may be even more interesting and challenging for students.

## II. Extracting the Information and Question

**Format:** whole group brainstorming, individual brainstorming, or think-pair-share.

List as mathematically and succinctly as possible the key information that may be useful in solving the problem and state what will count as an answer.

- Identify and list important information given in the problem:

What quantities are given?

What terms are important?

What constraints are given?

- Predict as much as you can about the final answer:

What will the units of the solution be (what will be counted or measured)?

What justification is needed/what am I trying to prove?

Can I figure out upper and lower bounds?

Could the answer be negative? Could it be a non-integer?

### Sample Activity: PoW IQ

“PoW IQ” stands for extracting the **I**nformation in the problem, and understanding the **Q**uestion.

Have each student make a table with very compact mathematical information from the problem in the left column, calculations or mathematical relationships they see in the middle column, and questions they should explore for the final answer in the right column.

#### Key Outcomes

- Student ownership and understanding of the constraints a full solution requires.
- Articulating mathematical information in a simple, compact format that makes patterns and relationships visible and moves students toward possible solution paths.

## III. Paraphrasing

**Format:** think-pair-share or students working individually.

The goal of paraphrasing a problem is to have students analyze the language of a problem and make clear the mathematics behind the problem situation. Some prompts you might use with students are:

- Identify the key words/phrases in the problem. How would you define them?
- How would you rewrite the problem?
- Same math idea/Different math story: How could you put the problem in a different scenario, while preserving the math behind the problem?

### **Sample Activity: In Your Own Words**

Read the problem as a group. Discuss what the words mean, make sense of the situation and of the question that must be answered. Put away the problem and any notes and individually paraphrase the problem, using one of the prompts above.

#### **Key Outcomes**

- Learn to make sure the problem makes sense to you and that you know what has to be figured out.
- Put your thoughts in writing so you can compare your thinking to the original problem statement and see what you may be missing or changing without realizing it.

## ***IV. Acting it Out***

**Format:** whole group with a few volunteers to act it out and the rest advising, or in small groups.

This approach often requires the most teacher/expert support to ensure the tools or manipulatives that support the investigation are available, and that students are making sure their modeling of the problem fits the necessary constraints. Some problems do not easily lend themselves to this approach. Acting the problem out would probably be used in tandem with one of the activities above, since it doesn't end in a written representation of students' understanding of the problem. Some examples include:

- Physically acting out the problem by using actual materials from the problem situation or using virtual manipulatives (You might act out a simpler version of the problem, for example, using smaller numbers).
- Drawing a rough sketch (This is different from drawing a picture as a strategy to solve the problem, since you know you might be drawing it imperfectly and less representative, but you are just trying to get a sense of relationships).
- Doing the problem "wrong": similar to a quick version of guess and check, doing the problem wrong can refer to working through the problem by guessing a number for an unknown quantity, or trying to find an answer that works without being sure you have found every possible answer. In either case, the focus is on understanding the problem scenario and key relationships, rather than trying to get a full solution to the problem.

### **Sample Activity: Try It!**

As a group, students and the teacher work together to decide on ways to represent the key objects or concepts in a problem. For example, the teacher might provide one or several virtual manipulatives or software tools, or the teacher might provide several types of physical manipulatives. The students then represent different object in the problem, and try to model the situation. Students in the "checker" role see if the model responds to the constraints of the problem that had been noticed and if not, suggest what might be wrong. As the students act out the problem, designated scribes add to the students' list of noticings/wonderings.

#### **Key Outcomes:**

- Use visual and physical intelligence to develop a sense of what is going on in the problem.
- Figure out an answer or a good estimate and use this to start thinking of explanations about why it works that way.